Sample Question Paper - 36
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. The sum of the 5^{th} and the 9^{th} terms of an A.P. is 30. If its 25^{th} term is three times its 8^{th} term, then find the A.P.

2. Find the mean of the following data:

Class	1-3	3-5	5-7	7-9
Frequency	12	22	27	19

3. If two cubes, each of edge 4 cm are joined end to end, then find the surface area of the resulting cuboid.

OR

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 42 cm and the total height of the vessel is 30 cm. Find the inner surface area of the vessel.

4. What is the distance between two parallel tangents to a circle of the radius 4 cm?

5. Find the value of f from the following data, if its mode is 65, where frequency 6, 8, f and 12 are in ascending order.

Class-interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	6	8	f	12	6	5

6. For what value of k, are the roots of the quadratic equation, $kx(x-2)+6=0$ equal?

OR

Solve for x : $\frac{16}{x} - 1 = \frac{15}{x+1}$; $x \neq 0, -1$

SECTION - B

7. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

8. The sum of four consecutive numbers in an A.P. is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

OR

The sum of the first three terms of an A.P. is 48. If the product of the first and second terms exceeds four times the third term by 12, find the A.P.

9. Draw a line segment $AB = 6.5$ cm and divide it internally in the ratio 3 : 5.

10. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

SECTION - C

11. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower at the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

OR

Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points. [Use $\sqrt{3} = 1.732$]

12. Raman made a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 21 cm and its length is 36 cm. If each cone has a height of 9 cm, find the volume of air contained in the model that Raman made.

Case Study - 1

13. An electric scooter manufacturing company wants to declare the mileage of their electric scooters. For this, they recorded the mileage (km/charge) of 50 scooters of the same model. Details of which are given in the following table.

Mileage (km/charge)	100-120	120-140	140-160	160-180
Number of scooters	7	12	18	13

Based on the above information, answer the following questions.

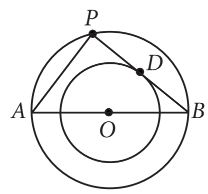
(i) Find the average mileage.

(ii) Find the modal value of the given data.

Case Study - 2

14. If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent.

(i) The diameter of two concentric circles are 10 cm and 8 cm. AB is the diameter of the bigger circle and BD is the tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of AP.



(ii) Two concentric circles are such that the difference between their radii is 4 cm and the length of the chord of the larger circle which touches the smaller circle is 24 cm. Then find the radius of the smaller circle.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Let a be the first term and d be the common difference of the A.P.

Now, $a_n = a + (n - 1)d$

$a_5 + a_9 = 30$ (Given)

$\Rightarrow a + 4d + a + 8d = 30 \Rightarrow 2a + 12d = 30$ ...(i)

Also, $a_{25} = 3a_8$

$\Rightarrow a + 24d = 3(a + 7d) \Rightarrow a + 24d = 3a + 21d$

$\Rightarrow 2a - 3d = 0$ ...(ii)

On solving (i) and (ii), we get $a = 3, d = 2$

$\therefore$ A.P. is 3, 5, 7, ...

2. The frequency distribution table from the given data can be drawn as :

Class	Class marks (x_i)	Frequency (f_i)	f_ix_i
1-3	2	12	24
3-5	4	22	88
5-7	6	27	162
7-9	8	19	152
		$\Sigma f_i = 80$	$\Sigma f_ix_i = 426$

$\therefore$ Mean $= \dfrac{\sum f_ix_i}{\sum f_i} = \dfrac{426}{80} = 5.325$

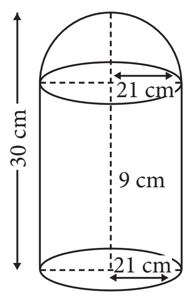
3. $\because$ Two cubes of edge 4 cm each are joined end to end to form a cuboid.

$\therefore$ For resulting cuboid, length $(l) = 4 + 4 = 8$ cm, breadth $(b) = 4$ cm and height $(h) = 4$ cm

$\therefore$ Surface area of cuboid

$= 2(lb + bh + hl) = 2(8 \times 4 + 4 \times 4 + 4 \times 8) = 160$ cm^2

OR



Radius of the hemisphere and cylinder, r

$= \dfrac{42}{2} = 21$ cm

Height of the cylinder, h

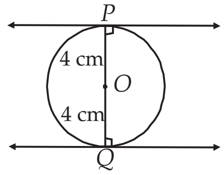
$= 30 - 21 = 9$ cm

$\therefore$ Internal surface area of the vessel = Curved surface area of cylinder + Curved surface area of hemisphere

$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times \dfrac{22}{7} \times 21(9 + 21) = 3960$ cm^2

4. We know that two tangents are parallel if and only if tangents are drawn at the end point of diameter.



So, PQ is a diameter of circle.

$PQ = 2$(radius of circle)

$= 2 \times 4 = 8$ cm

Hence, distance between two parallel tangents is 8 cm.

5. Here, given mode = 65, which lies in interval 60-80.

$\therefore$ Modal class is 60-80.

So, $l = 60, f_1 = 12, f_0 = f, f_2 = 6, h = 20$.

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow 65 = 60 + \left(\dfrac{12 - f}{2 \times 12 - f - 6}\right) \times 20 \Rightarrow 5 = \dfrac{12 - f}{18 - f} \times 20$

$\Rightarrow 90 - 5f = 240 - 20f \Rightarrow 15f = 150 \Rightarrow f = 10$

6. We have, $kx(x - 2) + 6 = 0$ or $kx^2 - 2kx + 6 = 0$

For equal roots, discriminant, $D = 0$

i.e., $4k^2 - 24k = 0 \Rightarrow 4k(k - 6) = 0$

$\Rightarrow k = 6$ ($\because k \neq 0$)

OR

We have, $\dfrac{16}{x} - 1 = \dfrac{15}{x + 1}$; $x \neq 0, -1$

$\Rightarrow \dfrac{16 - x}{x} = \dfrac{15}{(x + 1)} \Rightarrow (16 - x)(x + 1) = 15x$

$\Rightarrow 16x + 16 - x^2 - x = 15x$

$\Rightarrow 16 - x^2 = 0 \Rightarrow (4 - x)(4 + x) = 0$

$\Rightarrow x = 4$ or $x = -4$

7. Let the present age of the girl be x years.

$\therefore$ Present age of the girl's sister $= \dfrac{x}{2}$ years

After four years,

Age of girl $= (x + 4)$ years

Age of girl's sister $= \left(\dfrac{x}{2} + 4\right)$ years

Now, according to question, $(x + 4)\left(\dfrac{x}{2} + 4\right) = 160$

$\Rightarrow \dfrac{(x + 4)(x + 8)}{2} = 160$

$\Rightarrow x^2 + 8x + 4x + 32 = 2 \times 160$

$\Rightarrow x^2 + 12x + 32 - 320 = 0$

$\Rightarrow x^2 + 12x - 288 = 0$

$\Rightarrow$ $x^2 + 24x - 12x - 288 = 0$

$\Rightarrow$ $(x - 12)(x + 24) = 0$ $\Rightarrow$ $x = 12$ or -24

Since, age of the girl cannot be negative.

$\therefore$ $x = 12$

Thus, the present age of the girl is 12 years and that of her sister is $\frac{12}{2}$ years *i.e.*, 6 years.

8. Let the four consecutive numbers be $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$.

Sum of four numbers = 32 [Given]

$\Rightarrow$ $(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow$ $4a = 32 \Rightarrow a = 8$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow$ $\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$ $\Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$

$\Rightarrow$ $8a^2 = 128d^2$ $\Rightarrow$ $d^2 = \frac{8a^2}{128} = \frac{8 \times 64}{128} = 4$

$\therefore$ $d = \pm 2$

If $d = 2$, then the numbers are $(8 - 6)$, $(8 - 2)$, $(8 + 2)$ and $(8 + 6)$ *i.e.*, 2, 6, 10, 14.

If $d = -2$, then the numbers are $(8 + 6)$, $(8 + 2)$, $(8 - 2)$, $(8 - 6)$ *i.e.*, 14, 10, 6, 2.

Hence, the numbers are 2, 6, 10, 14 or, 14, 10, 6, 2.

OR

Let a and d be the first term and the common difference respectively of A.P..

Sum of first n terms is $S_n = \frac{n}{2}(2a + (n-1)d)$

So, $S_3 = \frac{3}{2}(2a + 2d) \Rightarrow 48 = 3(a + d)$

$\Rightarrow$ $a + d = 16$ ...(i)

Also, given $a_1 \times a_2 - 4a_3 = 12$

$\Rightarrow$ $a \times (a + d) - 4(a + 2d) = 12$

$\Rightarrow$ $a \times 16 - 4(16 + d) = 12$ (from (i))

$\Rightarrow$ $16a - 64 - 4d = 12 \Rightarrow 16a - 4d = 76$

$\Rightarrow$ $4a - d = 19$ ...(ii)

Solving (i) and (ii), we get

$5a = 35 \Rightarrow a = 7$

Put $a = 7$ in (i), we get $d = 9$

Thus, A.P. is 7, 16, 25, ...

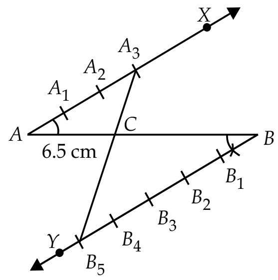
9. Steps of construction :

Step-I : Draw a line segment $AB = 6.5$ cm.

Step-II : Draw a ray AX making an acute angle with the line segment AB.

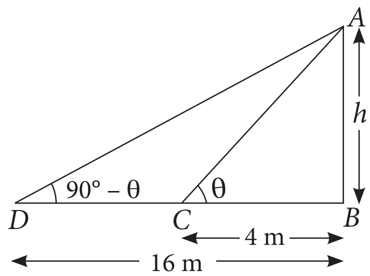
Step-III : Draw another ray $BY \parallel AX$ such that $\angle ABY = \angle BAX$.

Step-IV : Mark 3 points *i.e.*, A_1, A_2, A_3 on AX and 5 points *i.e.*, B_1, B_2, B_3, B_4, B_5 on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.



Step-V : Join A_3B_5 which intersects AB at point C. Thus, C divides AB in the ratio 3 : 5, *i.e.*, $AC : CB = 3 : 5$.

10. Let AB be the tower of height 'h' and C, D are points at a distance of 4 m and 16 m from the foot B respectively.



Let $\angle ACB = \theta$ $\Rightarrow$ $\angle ADB = 90° - \theta$

Now, in ΔACB

$\tan\theta = \frac{AB}{BC} = \frac{h}{4}$ ...(i)

In ΔADB, $\tan(90° - \theta) = \frac{AB}{DB} = \frac{h}{16}$

$\Rightarrow$ $\cot\theta = \frac{h}{16}$ ...(ii)

Multiplying (i) and (ii), we get

$\tan\theta \times \cot\theta = \frac{h}{4} \times \frac{h}{16}$

$\Rightarrow$ $\tan\theta \times \frac{1}{\tan\theta} = \frac{h^2}{64}$

$\Rightarrow$ $h^2 = 64$ $\Rightarrow$ $h = 8$ m

11. Let AB be the hill and CD be the tower of height 50 m.

Now in ΔCBD, $\tan 30° = \frac{CD}{BD}$

$\Rightarrow$ $BD = \frac{CD}{\tan 30°}$ $\Rightarrow$ $BD = \frac{50}{\left(\frac{1}{\sqrt{3}}\right)}$

$\Rightarrow \; BD = 50\sqrt{3}$ m

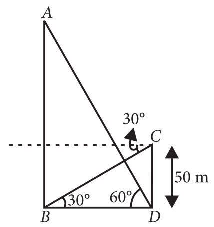
In ΔABD, $\tan 60° = \dfrac{AB}{BD}$

$\Rightarrow \; AB = BD \times \sqrt{3}$

$= \left(50\sqrt{3} \times \sqrt{3}\right)$

$= 50 \times 3 = 150$ m

Hence, the height of the hill is 150 m.



OR

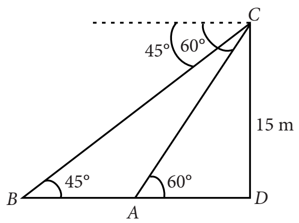
Let CD be the tower.

In ΔACD,

$$\tan 60° = \frac{CD}{AD} = \frac{15}{AD}$$

$$\Rightarrow \; \sqrt{3} = \frac{15}{AD}$$

$$\Rightarrow \; AD = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ m}$$



In ΔBCD,

$$\tan 45° = \frac{CD}{BD} = \frac{15}{BD} \;\Rightarrow\; 1 = \frac{15}{BD} \;\Rightarrow\; BD = 15 \text{ m}$$

Distance between A and B = $AB = BD - AD$

$= 15 - 5\sqrt{3} = 15 - 8.66 = 6.34$ m

12. Radius of each cone (r) = Radius of cylinder (r)

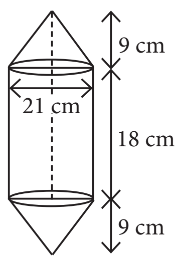
$= \dfrac{21}{2} = 10.5$ cm

Height of each cone (h) = 9 cm

$\therefore$ Height of cylindrical portion (h_1)

$= 36 - 9 - 9 = 18$ cm

$\therefore$ Volume of the air contained in model = Volume of cylindrical part + 2 × Volume of cone



$$= \pi r^2 h_1 + 2 \times \frac{1}{3}\pi r^2 h = \pi r^2\left[h_1 + \frac{2}{3}h\right]$$

$$= \frac{22}{7} \times (10.5)^2\left[18 + 2 \times \frac{1}{3} \times 9\right]$$

$$= \frac{22}{7} \times (10.5)^2\left[18 + 6\right] = \frac{22}{7} \times 110.25 \times 24 = 8316 \text{ cm}^3$$

13. Given frequency distribution table can be drawn as :

Class interval	**Class mark (x_i)**	**Frequency (f_i)**	$x_i f_i$
100-120	110	7	770
120-140	130	12	1560
140-160	150	18	2700
160-180	170	13	2210
Total		50	7240

(i) Clearly, average mileage

$= \dfrac{7240}{50} = 144.8$ km/charge

(ii) Since, highest frequency is 18, therefore, modal class is 140-160.

Here, $l = 140, f_1 = 18, f_0 = 12, f_2 = 13, h = 20$

$$\therefore \; \text{Mode} = 140 + \frac{18-12}{36-12-13} \times 20 = 140 + \frac{6}{11} \times 20$$

$$= 140 + \frac{120}{11} = 140 + 10.91 = 150.91$$

14. (i) Here, in right ΔOBD, OB = 5 cm and OD = 4 cm.

$\therefore \; BD = \sqrt{25-16} = \sqrt{9} = 3$ cm

Since, chord BP is bisected by radius OD.

$\therefore \; BP = 2BD = 6$ cm

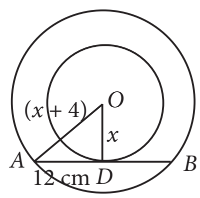
Also, $\angle APB = 90°$ (Angle in a semicircle)

$\therefore \; AP = \sqrt{AB^2 - BP^2} = \sqrt{100-36} = \sqrt{64} = 8$ cm

(ii) Let x be the radius of smaller circle.

Now, $OA^2 = OD^2 + AD^2$

$\Rightarrow \; (x + 4)^2 = x^2 + 12^2$



$\Rightarrow \; 8x + 16 = 144$

$\Rightarrow \; x = 16$ cm